Instructions: Try these problems from the textbook as practice (answers below).

Problems

19-30. Use cylindrical coordinates to evaluate the following integrals.

19. $\iiint_E \sqrt{x^2+y^2}\ dV$, where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$.
21. $\iiint_E (x+y+z)\ dV$ where E is the solid in the first octant that lies under the paraboloid $z=4-x^2-y^2$.
23. $\iiint_E x^2\ dV$ where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$.

25-27. Compute the volume of the solid...

25. enclosed by the cone $z=\sqrt{x^2+y^2}$ and the sphere $x^2+y^2+z^2=2$.
27. that lies between the paraboloid $z=24-x^2-y^2$ and the cone $z=2\sqrt{x^2+y^2}$.

31. Evaluate the integral $\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\ dz\ dx\ dy$ by changing to cylindrical coordinates.

Answers

19. 384π
21. $\dfrac{8\pi}{3} + \dfrac{128}{15}$
23. $\dfrac{2\pi}{5}$
25. $\dfrac{4}{3}\pi(\sqrt{2} - 1)$
27. $\dfrac{512\pi}{3}$
31. 0